The Emergence of Consciousness

A Scientific and Philosophic Overview

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Abstract
The notion of emergence has gained much popularity and acceptance in contemporary philosophy and science. It is the aim of this paper to examine and contrast some of the more influential work coming within these disciplines and to modestly consider whether we are yet on the road to an emergentist science of consciousness.

Introduction

The emergence reduction debate began around two and a half thousand years ago when Aristotle, in opposition to Democritus, proposed that "*the whole is greater than the sum of its parts*". Alongside his tutor, Plato also vehemently denied the notion that atomism could ever explain the sheer wealth of beauty on Earth; he asked simply, "*is the world created or uncreated*". With such a pedigree of influential power, this Aristotelian school of thought dominated western philosophy and science for over 1600 years with atomism sidelined as a mere footnote of curiosity.

In the late 15th century, however, Europe was swept up by the reconnection to classical antiquity instantiated by the Italian Renaissance and this new knowledge rooted at distributed by the advent of the printing press; by the 16th century a scientific revolution had begun.

In England, the groundswell of atomism began with Northumberland circle led by Henry Percy. Their influence replanted the atomic seed which soon spread through the elite of the British scientific community, including Sir Francis Bacon. At around the same time, Galileo Galilei also began to tackle some of the basic problems of Aristotelian physics and within The Assayer described a more reductionist view of all phenomena as "*matter in motion*".

However, the full revival of atomism came in the late 16th and early 17th century with the rise of the mechanical philosophy championed by Gassendi and Newton and made acceptable by Descartes. Newtonian mechanics and the atomistic worldview struggled with its inherent atheism in a world of religion. But in 1618, Rene Descartes, influenced by amateur scientist Isaac Beeckman, promoted the less onerous notion of material "*corpuscles*"; a very similar notion to more fundamental atomism championed by Gassendi but with room for a separate realm of existence for thought, soul and importantly god. Dualism was born and Newtonian mechanics received acceptability alongside the religious doctrine.

Over the next 300 hundred years, mechanism and atomism, the twin towers of the reductionist worldview, catalyzed their own growth by fueling the new economy of technological advancement. Sociology, psychology, biology and chemistry became mere extensions to the all powerful core of physics; the only true realm of causal efficacy and physical reality. The whole, it seemed, was entirely constituted from the mechanical laws of its fundamental parts.

Science itself became dominated by reductionist methodology; physics and chemistry reduced matter to atoms; biology reduced life to DNA and the cognitive sciences reduced mind to brain. As religion lost its grip on the social domain, science became free to adopt a more fundamentally atomistic philosophy; materialism and physicalism began to dominate both the scientific and philosophical literature and, more importantly, their culture.

As ever though, nature was not so easily tamed.

In the mid 1900s the pendulum began to turn once more; and, as with so many important cultural paradigms, the change sprang up independently in disparate areas to together drive the momentum for the shift.

The most damaging blows to reductionism came most surprisingly, but with a refreshing acceptance, from within physics herself. First, the material world and the world of thermodynamics were wondrously and beautifully linked with Einstein's famous equation. Matter was shown to be interchangeably reducible to energy and the fundamental atoms that Gassendi and Democritus had championed were eliminated by the force of action.

Secondly, and with a little more reluctance, came the related mechanics of this new quantum reality. The deterministic mechanics of Newtonian physics were slowly replaced with the new probabilistic laws of Heisenberg Uncertainty. Nature, it seemed, was inherently stochastic (Heisenberg 1944; Schrödinger 1926).

Disparately, and at the same time, the advent of the computer was propelling mathematics into previously unreachable worlds for experimentation. Non-linear, multi-dimensional and dynamic systems were for the first time being explored and a new science was slowly emerging with a tentative new worldview of nested thermodynamics (Lorenz 1963; Van Bertalanffy 1969; Prigogine 1981; Kauffmann 1993).

The invisible shockwave rolled up through the entire scientific doctrine, slowly loosening many of the reductionistic shackles.

Biologists quickly began to define life in systemic terms, rather than the selfish mechanics of DNA (Maturana & Varella 1980). Newly revised notions of self-organisation, and dissipative structure began to reinforce the systemic notions of autonomy and control; blossoming with the much needed re-vitalisation of planet Earth herself (Lovelock 1979).

Computationalism itself breathed fresh life into the philosophy of mind; for the first time the Cartesian gap between the logical manipulation of information and the material world was demonstrably bridged. However, the nested world of thermodynamics is beginning to slowly mold further, more holistic changes in attitude.

The notion of emergence is once again gaining popularity and acceptance in philosophy and science; and this paper aims to examine and contrast *some* of the more influential work coming within from both. For philosophers, emergence within an ontology of nested systems might hold the key to explaining consciousness and the autonomy of mind; for scientists, the notion is a little more grounded in its attempt to define the process by which order arises within systems where once it was not. We shall briefly examine both in a modest attempt at establishing whether we are yet on the road to an emergentist science of consciousness.

Philosophic Emergence of Consciousness

The notion of an "*emergence of consciousness*" first resurfaced within philosophy in the early 1900s with the British Emergentists; J.S. Mill, C.D. Broad and Samuel Alexander. Although different in detail, common to all three was a layered view of the world. Unique strata of complexity were said to lay nested amongst one another; starting with the fundamental strata of the physical, following by the chemical, the biological, the psychological and finally the social.

For Mills (1843), every particle *p* in a particular strata, would be acted upon by a combination of heteropathic causes (higher strata) and homopathic causes (same or lower strata) in a kind of "*composition of cause*", much like the composition of forces in Newtonian physics. Within this view, causal efficacy would be found acting from within all strata and would not be unique to the physical world. The emergent consciousness, accordingly, would be perceived as a proponent of such causal power.

Broad's (1925) view, in contrast, placed a stronger emphasis on the emergence of properties as well as causality. For Broad, each stratum contained a set of irreducible properties that *emerged* from lower level properties by trans-ordinal laws. Through the study of the lower level properties alone, it would be impossible to predict the presence of high level phenomena, unless you are aware of the trans-ordinal laws; and you can only become aware of the trans-ordinal laws if you were first to observe the higher level phenomena. The causal efficacy of Broad's model is unclear, although it is said that he highlights the existence of higher level causal interactions; and as such it seems that in this view a "*consciousness*" would be a metaphysically emergent collection of properties and efficacious causal interactions.

Finally, Samuel Alexander's notion of emergence (Alexander 1920), inspired by the work of C. L. Morgan (later published in 1923), proposes that higher level qualities exist (with their own behavioral laws) but that they are simply macroscopic patterns running through lower level microscopic interactions. Or in other words, there is a single substrate through which different behavioural stories may be told, dependant upon your choice of scale, but true causal efficacy is only found at the microscopic layer. This view is by far the closest related, to the contemporary view of a philosophic emergence of consciousness.

It is somewhat interesting to note that these views all came before the major shift in reductive ontology and that they were, for the most part, all dismissed until the recent post-Turin revival of the mind-body problem.

Since Descartes introduced the dualistic split between mind and body in the 17th century, many attempts have been made in closing the explanatory gap. Numerous philosophical stances have been offered with a varying degree of acceptance and in the last 50 years emergence has played an increasing important role in a great many of those. It is now quite common for the emergence of consciousness to play a key role for both the dualist and the monist camps; although admittedly from somewhat different perspectives.

Emergentist monists would cite emergence in a way similar to Alexander; a single substrate (monism) through which many causal patterns can be drawn. One pattern is not reducible to the other, and neither holds greater causal efficacy over the other; both simply exist and within the same physical substrate. This notion is interchangeably referred to as anomalous monism (Davidson 1970) non-reductive physicalism and the woven view of causality is referred to as supervenience[1] (Kim 1993).

For emergentist dualists, however, when matter is organised in an appropriate way (i.e. in the way that human bodies are organised) mental properties will emerge. For some, these properties are ontologically distinct, but causally inefficacious (epiphenomenalism), for others however, the emergence of consciousness properties brings with it a new strain of causal power (interactionism or parallelism); and in an effort to establish this causal efficacy for the emergent mind, a new notion of downwards causation has been proposed (Campbell D.T. 1974, 1990; Campbell R.L. & Bickhard 2002; Bickhard & Campbell D.T. 2000; Emmeche, Køppe & Stjernfelt 1990).

In downward causation, a higher-level (emergent) phenomenon has a causal influence on the lower-level particles; the proffered example being the candle flame as described by Campbell and Bickhard (2002). The emergent flame (the process of burning) is a dissipative process which maintains its own existence through the radiation of convection currents in order to draw in the fuel (oxygen) that it requires. Without the underlying physical presence of the correct substrate (oxygen, wax) the flame could not exist; and without the emergent flame, the substrate would not be drawn. Similarly, evolution is defined as a prominent, although admittedly controversial, example of such downward causation.

In summary then, it appears that the notion of emergence is used philosophically to describe many different aspects of consciousness. For some dualists it is considered an adequate means of describing the separation between the ontological realms of mind and matter; whilst for others, including many monists, it is a means of adequately explaining higher levels of causal efficacy. For a scientist, however, the notion of emergence is slowly being formalised as a highly specific phenomenon; and the question is whether these two worldviews can be combined to provide a scientifically grounded perspective of the philosophic emergence of consciousness.

Scientific Emergence

The revival of scientific emergence, as with much of the post-modern world, has its roots buried deep inside the pioneering mathematics of the 2nd World War. Alan Turing's monumental reformulation of Gödel's universal language of arithmetic brought forth the mathematics of computation theory. At the same time, on the other side of the pond, Norbert Weiner and Claude Shannon's work on control and information theories reintroduced Leibniz's sociological concepts of communication. The traditionally human domains of knowledge, logic, and communication were beginning to be mechanised and the previously ethereal realms of thought and mind were, for the first time, becoming legitimate subjects for scientific enquiry (see Heims 1980)..

Soon after, and through the embracement of these new ideas, John Von Neumann began his

[1] It should be noted that supervenience is also often stated in such a way that mental causality supervenes *over* the physical causality with the subtle implication that the physical world still holds full efficacy. However, this should not necessarily be the case as otherwise supervenience can simple be considered as a monist epiphenomenalism.

illustrious work on the architectural mechanics of the early computers; but he also, somewhat less famously, followed Turing into the embryonic field of theoretical biology. Here, whilst Turin published many important papers on the subject of morphogenesis, Von Neumann settled down to tackle the problem of self-replication. During this period, and with his colleague Stanislaw Ulam, Von Neumann introduced the notion of a multi-dimensional lattice of cells with a finite state automata lying within each cell. This was the birth of the cellular automata (see Heims 1980).

Nearly thirty years later, John Conway took a fascination in von Neumann's cellular automata and in 1970, using just a chessboard and set of draughts, he devised and published a curious set of automata rules known as "*the Game of Life*". It was these rules that would hold the key to a new kind of science; a science of emerging complexity.

Initially, the Game of Life gained interest purely because of the visual semblance which the activation patterns had with to the world of bacterium; to an onlooker, the movement of draughts pieces in a CA running the Game of Life looked almost alive. However, Conway, with the assistance of the mathematicians Elwyn Berlekamp and Richard Guy, later went on to prove that the these activations, given a large enough chess board, and the correct starting position, were, in theory, universally computable, just as a Turin machine (Berlekamp, Conway & Guy 1982). In other words, a CA running the Game of Life could be used as a computer running any imaginable computer program.

Imagine on one level, a collection of simple automata, each one blindly turning itself on and off based on the same set of simple rules (e.g. "*if 3 of my neighbours are on, then I'll stay on, otherwise, I'll go off*"). But, these same simple parts, when viewed as a whole, are running a flight simulator, or translating the entire works of William Shakespeare into French.

Fascinated by this, Stephen Wolfram began to study the entire rule-space of cellular automata; looking for other interesting rulesets similar to the Game of Life; a rule-space that transpired to be rather large. As an example, consider Von Neumann's checkerboard CA. Each automaton would examine its surrounding 8 cells; so 9 including itself. As there each cell can either be on or off (2 states) then there are $2^9 = 512$ possible rules in the automaton's finite state rule-table. As each of these states will lead an automaton to become either on or off (2 states) there are 2^{512} possible rule-tables. In other words there are 10^{153} possible CA with a von Neumann configuration. Wolfram quickly realised that a larger dimensionality (a bigger neighbourhood) or a larger number variety of states (other than just on or off) would have an astronomical different on the rule-space he wanted to search.

He was left with no choice but to narrow his survey to the much smaller world of 1 dimensional (3 cell neighbourhood) binary (2 state) CA.; and here, there are just 256 rule-tables to examine. From this much more manageable position, he outlined a classification of the types of rules that could be found.

About half of the rule-tables tended to display either boring frozen static states (class 1) or periodic repetitive patterns (class 2) and the other half tended to result in random looking, patternless displays (class 3). However, Wolfram found that a tiny handful of rule-tables seemed to show "*interesting*" or "*complex*" patterns of behaviour (class 4) which didn't find neatly into any of the other classes (Wolfram 1984).

Chris Langton of the Sante Fe institute took this work a step further. By this time Dynamical Systems were already beginning to be studied as a mathematical theory of there own right. Using the language of chaos and attractors, Langton re-organised Wolfram's classes into *ordered behaviours* (class 1 & 2), *complex behaviours* (class 3) and *complex behaviours* (class 4); and described this region of complexity as a kind of "phase transition" between order and chaos (Langton 1989).

In the mid-to-late 1980s, Langton's terminology attracted a lot of wide-eyed attention and the complex region of dynamical space was soon romantically coined "*the edge of chaos*" (Packard 1988); however, this wonderful term is slowly becoming more and more estranged as scientific exploration of the new landscape gains a stronger footing. Recent work is beginning to re-describe this unique class of dynamical system as acting both in an ordered *and* in a chaotic fashion (Wuensche 1997; James 2005, 2006). More precisely, these systems appear to contain chaotically behaving parts, but yet orderly behaving wholes. For the first time, mathematics is beginning to study wholes which truly are greater than the sum of their parts.

We should, however, heed the words of John Holland in his exhortative book on this exciting new science; he writes, "*despite its ubiquity and importance, emergence is an enigmatic, recondite topic, more wondering at than analysed… it is unlikely that a topic so complicated will submit weakly to concise definition*" (Holland 1998).

Scientific Emergence of Consciousness

Robert Van Gulick recently published an interestingly concise overview of philosophic emergence in the Journal of Consciousness special issue, "The Emergence of Consciousness". Within this article he highlights the fact that "*speakers in the reduction debate often talk past one another by failing to distinguish ontological from representational notions, especially in interdisciplinary settings that combine scientists and philosophers*". He later goes on to define *metaphysical emergence* and *epistemological emergence* (Van Gulick 2001).

He states that metaphysical emergence can in be bisected into the *emergence of properties* and the *emergence of causal powers*; each of which can then be stated as having varying strength of flavour (from *special-kind*, through *modest-kind* to the extremist, *Radical-kind*).

He states, also, that epistemological emergence can be split into predictive and representational emergence, where; predictive or explanatory emergence describes a phenomenon which cannot be explained or predicted by the features or interactions of its parts; and representational emergence describes a phenomenon which cannot be described, or represented using the framework or language of its parts.

This split between the metaphysical and epistemological describes, rather well, our split between the philosophic and scientific notions of emergence. However, one must question the usefulness of such a discriminatory wedge between these two equally important disciplines. It certainly helps to stop them from "*talking past one another*", but it seems also, more importantly, to stop them from *talking to* one another.

The science of complexity should not be too swiftly belittled to be a mere mathematical curiosity. Dynamical systems theory is more than just a branch of mathematics, it is a burgeoning worldview based on the ontology of nested systems. In conjunction with the revitalised physics of thermodynamics, dynamical systems theory is beginning to tackle some of the many questions posed by the quantum destruction of atomism and Newtonian mechanics; and for many, complexity theory and scientific emergence lie at the very heart of this exciting new worldview. From within the systemic paradigm there is only structure and it carries on "*all the way down*".

From the perspective of the philosophical monist this worldview would mean that the single substrate, from which all things are made, must be structure or organisation; and from complexity theory, as we have seen, new structure can emerge at new scales. For our monist then, all manner of things must be able to emerge; and this in turn must include all mental properties. Within this view, the Alexandrian model of causality can also be adopted; without having to grant causal efficacy to a basic physical stratum the many scales of behaviour can all freely interact in their own causal stories.

For the ontological dualist the systemic worldview and complexity theoretic definition of emergence are less helpful; but this may not be so entirely necessary. It has been suggested that if, from the systemic perspective, one were asked "are you a dualist", the correct answer would be "at the very least" (Rockwell 1995). This is because, for some, the nested world of system can be equally conceived as a pluralist ontology. In fact, the distinction between a pluralist and monist in this view is simply whether a higher realm of structure is considered a new substance or not.

The real *hard* problem for a scientific definition for the emergence of consciousness lies in the nature of phenomenology (a subject to deep and quarrelsome to go into in any real depth within this paper). Although linked very closely with the mind-body problem, the question of "*what it is like*" is more than just a question of causal efficacy and property creation. No current scientific theory of emergence even begins to tackle the problem of first-person perspective; and this is likely what David Chalmers means (Chalmers 1995). An objective science of atomism and mechanism, or of causality and properties, is simple inadequate to tackle the subjective world of experience, we are missing a third aspect. Until that gap is closed, a science of emergence will never fully satisfy all of the problems of consciousness.

Closing

A contemporary philosophy of emergence is very young and its science is even younger. Many exciting new discoveries have been made in recent years and it certainly seems that the pendulum has turned a little in the great old debate. The question now, however, is how far will it swing, and how much will it help us to answer our questions regarding our most beautiful and precious commodity; our consciousness.

Phenomenology still appears tantalisingly out of reach, but who knows what tomorrow might bring. The last 50 years has already seen one dramatic shift; and from the perspective of the parts one simply cannot predict what further global shifts are in store for us in the future.

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